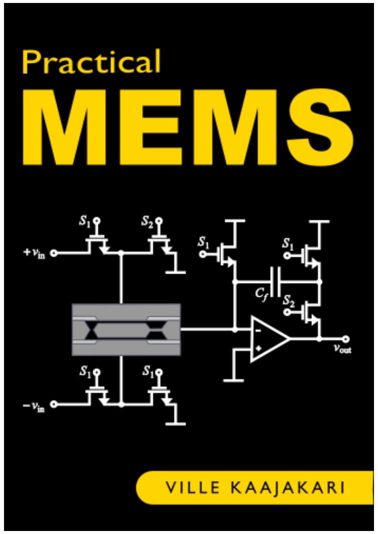
Practical MEMS

Chapter 3: Accelerometers

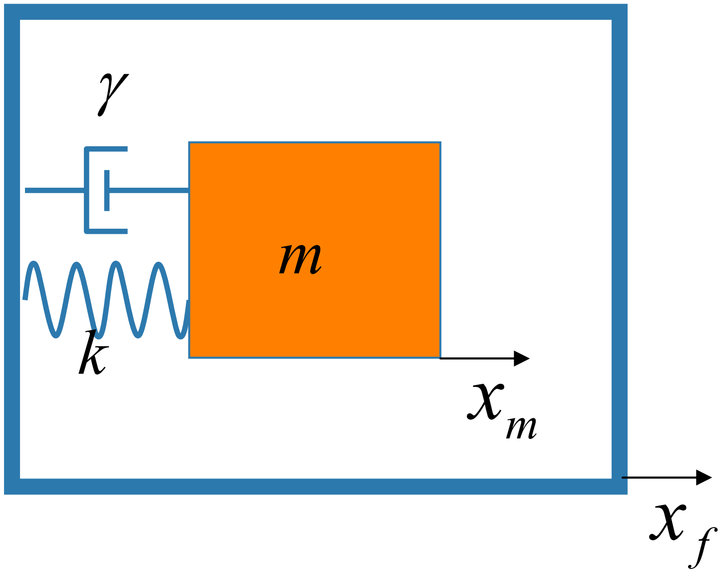


http://www.kaajakari.net/PracticalMEMS

Micromachined accelerometers

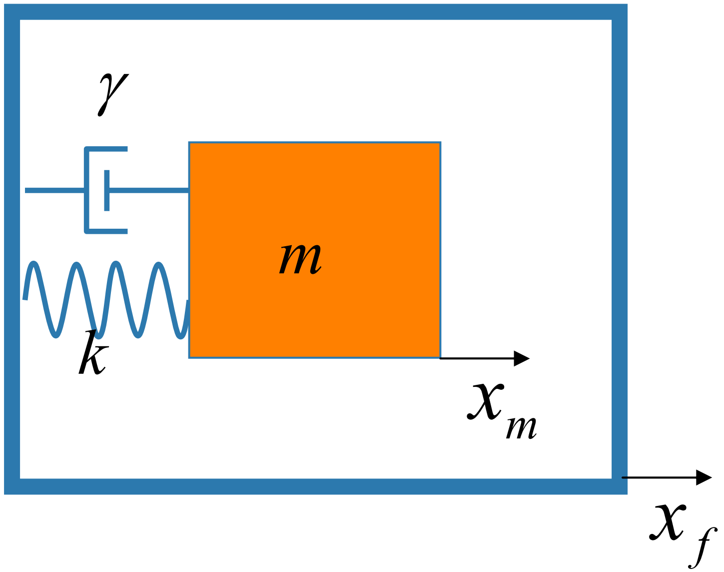
- The “second” MEMS product (first was pressure sensors).
- Applications:
  - Crash detectors for air bag deployment. Over 6,000 lives saved in US.
  - Low-G sensors are used for active suspensions and vehicle stabilization controls.
  - Motion based user interfaces (e.g. game consoles, cell phones)
  - Step counters, running speed and distance.
  - Digital cameras to determine the picture orientation.
  - Free fall detection to protect laptop hard drives

Principle of operation



- A proof mass is attached to frame with a spring.
- When the frame is accelerated, the proof mass follows the frame motion with a lag.

Frequency response



$$x = x_f - x_m$$

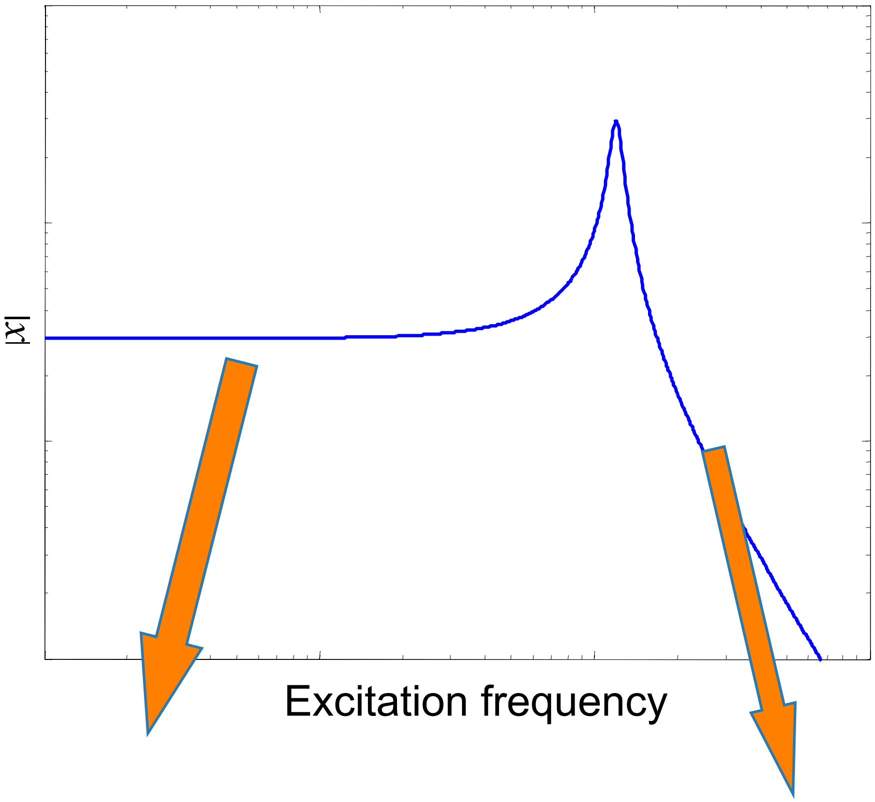
Equation of motion:

$$m\frac{\partial^2 x}{\partial t^2} + \gamma\frac{\partial x}{\partial t} + kx = m\ddot{x}_f$$

$$\Rightarrow \quad x = \frac{m\ddot{x}_f}{s^2 m + s\gamma + k}$$

The proof mass displacement relative to the frame is proportional to the acceleration!

Amplitude of the response



$$|x| = \frac{m\ddot{x}_f / k}{\sqrt{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + \left(\frac{\omega}{Q\omega_0}\right)^2}}$$

where $\omega_0 = \sqrt{\frac{k}{m}}$ and $Q = \frac{\omega_0 m}{\gamma}$

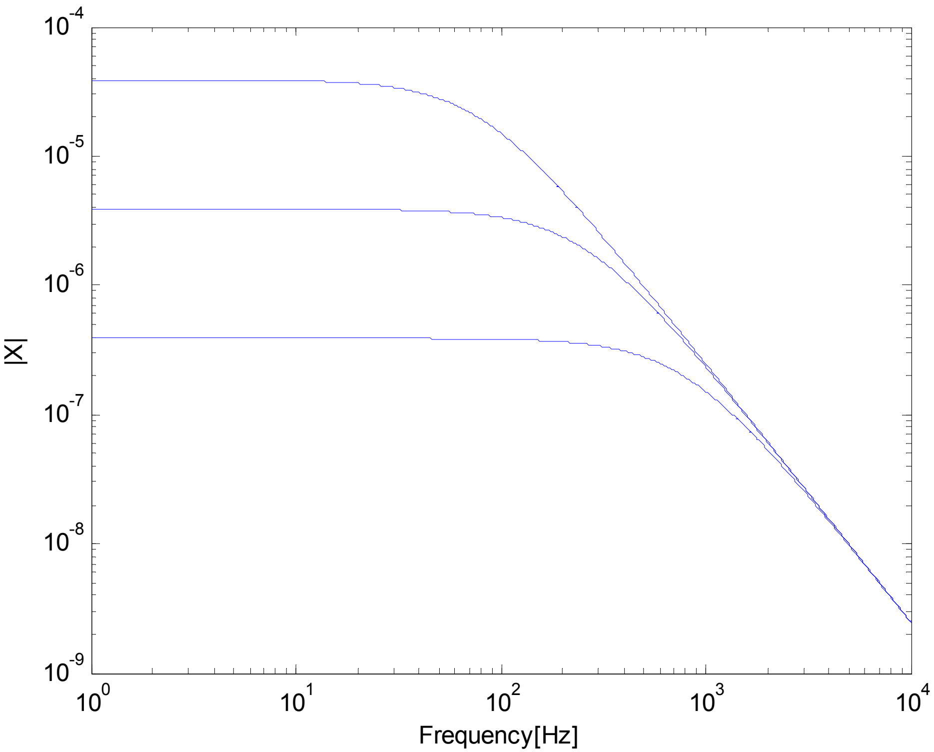
low frequency response

$$|x| \approx \frac{m\ddot{x}_f}{k} = \frac{\ddot{x}_f}{\omega_0^2}$$

high frequency response

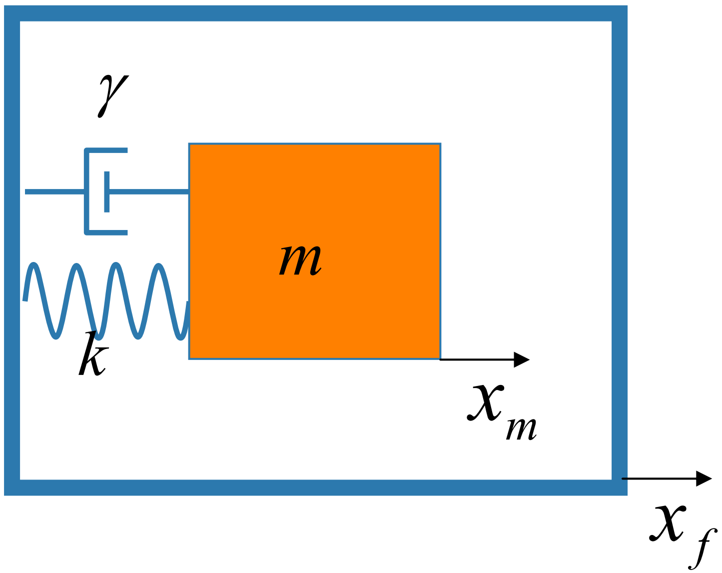
$$|x| \approx \frac{\ddot{x}_f}{\omega^2} = x_f$$

Changing the resonant frequency



- Lowering ω_0 improves low frequency response but does not affect high frequency response.
- Overall, lowering ω_0 helps.
- Taken to extreme $\omega_0 = 1\text{-}2$ Hz! (Macroscopic seismometers).
- For MEMS typically $\omega_0 > 50$ HZ.

Scaling laws for accelerometers



$$x_f - x_m = \frac{m}{k}\ddot{x}_f$$

What happens when all dimensions are reduced 10x?

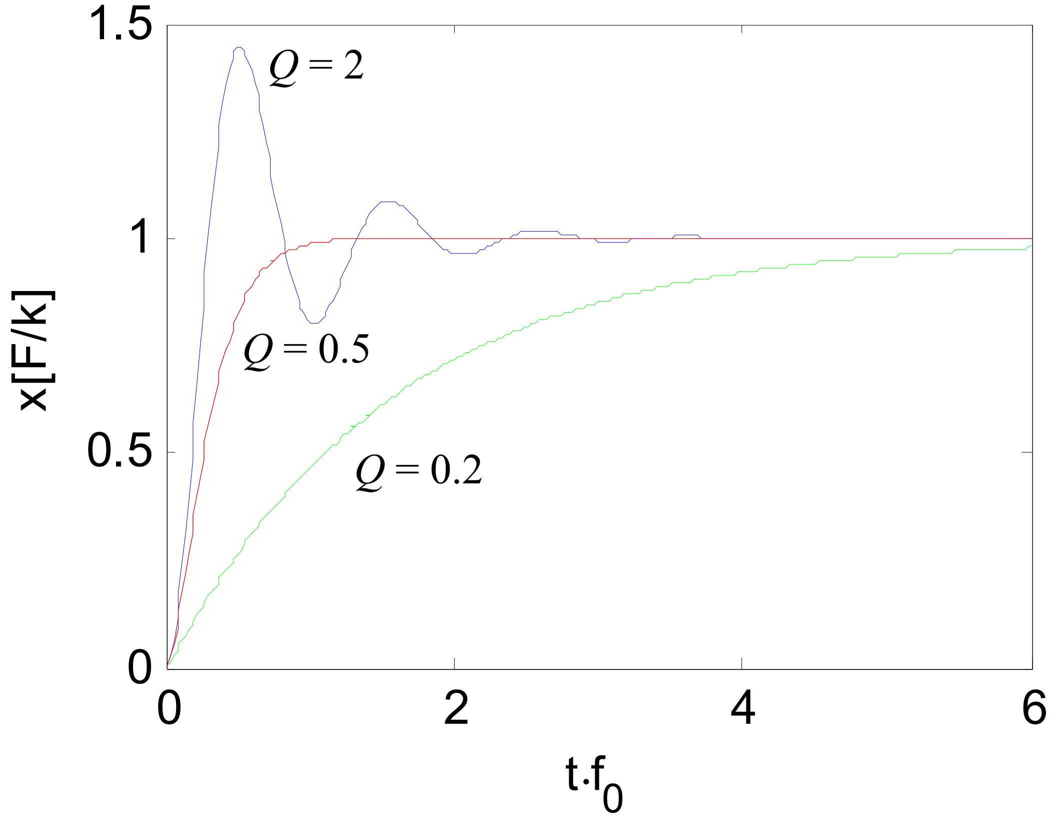
$$m \rightarrow \frac{m}{1{,}000}$$

$$k \rightarrow \frac{k}{10}$$

$$x_f - x_m \rightarrow \frac{x_f - x_m}{100}$$

(Analog devices accelerometers measure 0.1 Å displacements!)

Time domain response

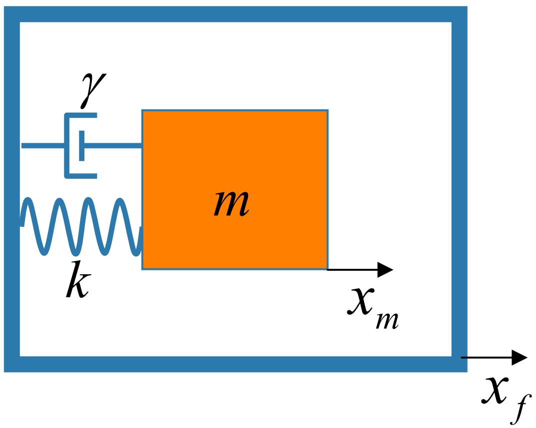


Critical damping or over damping preferred for clean response

Sensing principles

- Piezoresistive
  - Stress sensitive resistors integrated in the springs
  - Robust
  - Noisy, high power, and large temperature dependency
- Capacitive
  - Direct measurement of displacement
  - Low power, low noise
  - Small capacitance measurement is difficult
- Piezoelectric
  - Self generating
  - Signal proportional to change in stress – no dc signal!
- Magnetic
- Optical

Noise equivalent acceleration (spectral density)



Equate noise force generator and apparent acceleration force :

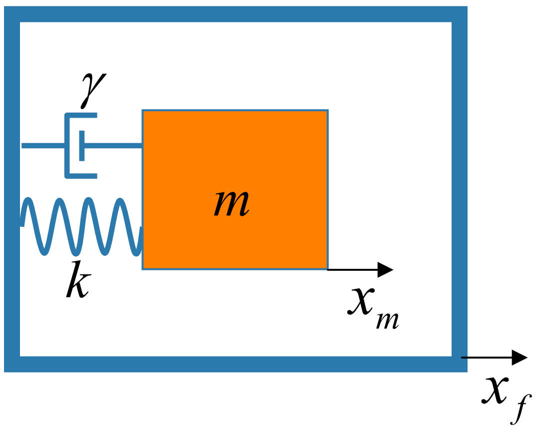
$$\overline{F}_n = \sqrt{4k_B T\gamma}$$

$$F_{\ddot{x}} = m\ddot{x}$$

Noise equivalent acceleration :

$$\overline{\ddot{x}_n} = \sqrt{\frac{4k_B T\omega_0}{mQ}} \quad \left[\mathrm{m/s^2/\sqrt{Hz}}\right]$$

Noise equivalent acceleration (rms acceleration)



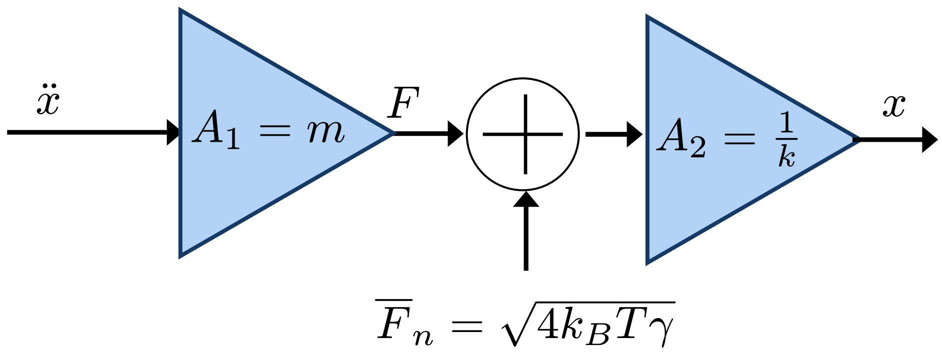
(1)
$$\frac{1}{2}kx_{\text{rms}}^2 = \frac{1}{2}k_B T$$
$$\Leftrightarrow x_{\text{rms}}^2 = \frac{k_B T}{k}$$

(2)
$$x = x_f - x_m = \frac{m}{k}\ddot{x}_f$$

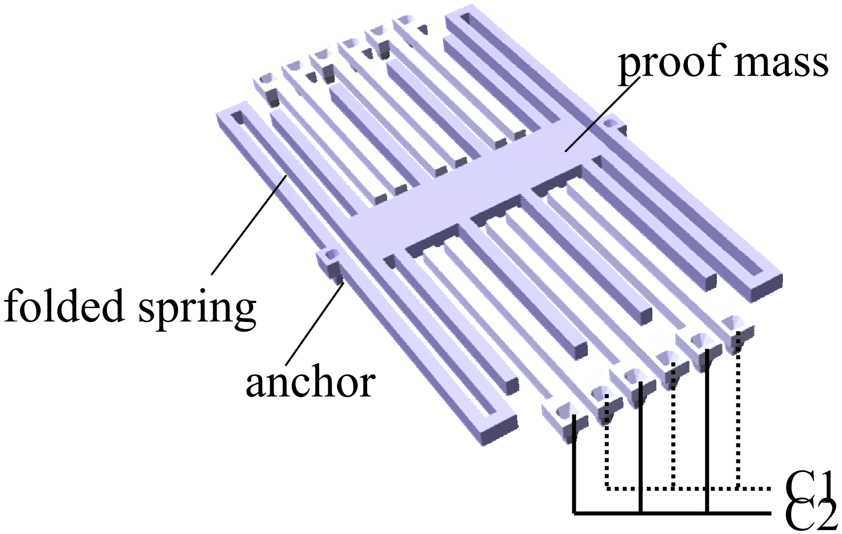
$$\Rightarrow \quad \ddot{x}_{\text{rms}} = \sqrt{\frac{k k_B T}{m^2}} = \sqrt{\frac{\omega_0^2 k_B T}{m}}$$

Total rms noise depends only on mass and resonant frequency!

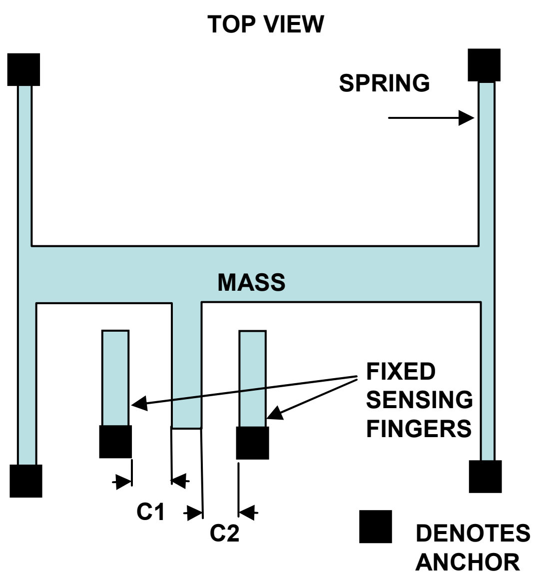
System level noise model

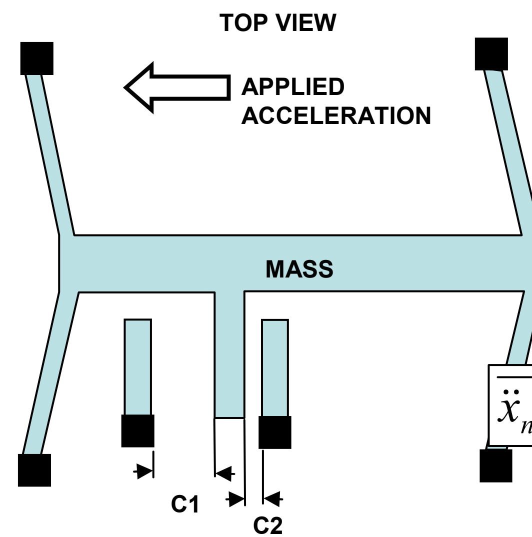


Surface micromachined accelerometer



Parameter	Symbol	Value	Units
Resonant frequency	f_0	22	kHz
Mass	m	0.1	nkg
Spring constant	k	2	N/m
Electrode capacitance	C_0	0.1	pF
Quality factor	Q	3-4	

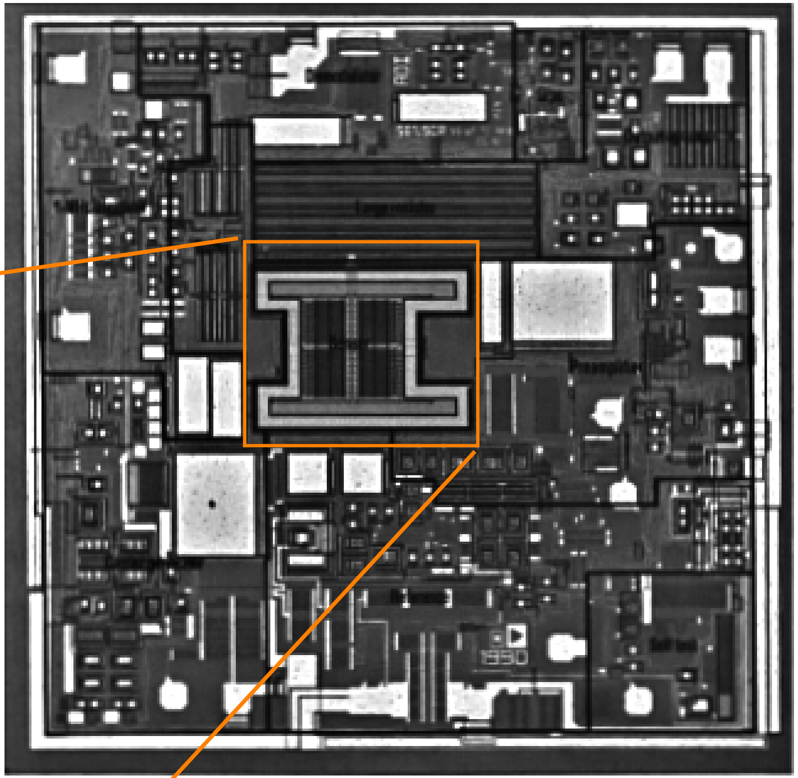


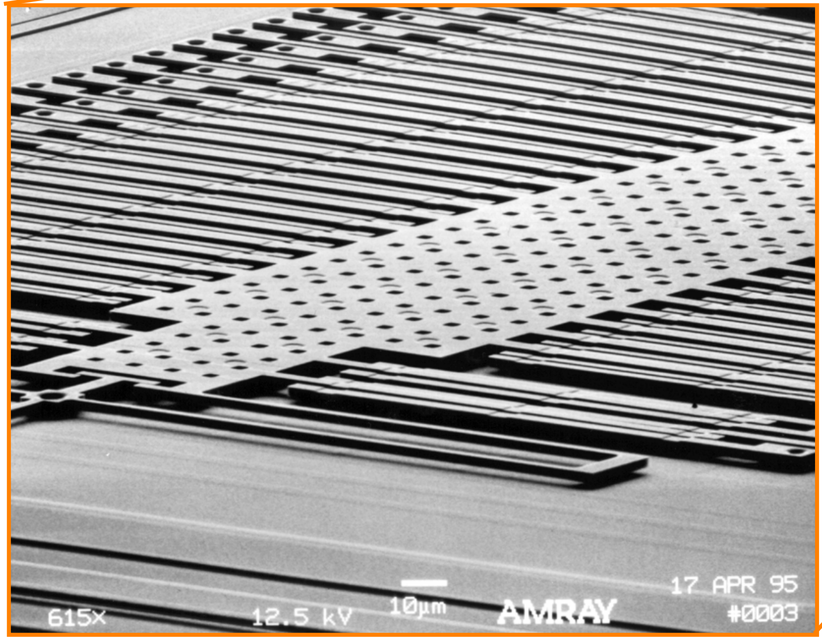


$$\overline{\ddot{x}_n} = 0.2\,\mathrm{mG}/\sqrt{\mathrm{Hz}}$$

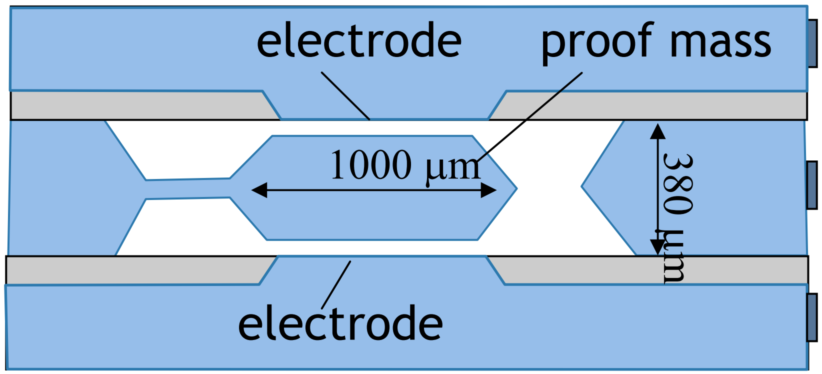
ADXL50 Accelerometer

- +/-50G
- Polysilicon MEMS & BiCMOS
- 3x3mm die





Bulk micromachined accelerometer



Parameter	Symbol	Value	Units
Resonant frequency	f_0	1	kHz
Mass	m	1	μkg
Spring constant	k	50	N/m
Electrode capacitance	C_0	5	pF
Quality factor	Q	0.1	

Noise equivalent acceleration :

$\overline{\ddot{x}_n} = 3\ \mu\text{G}/\sqrt{\text{Hz}}$